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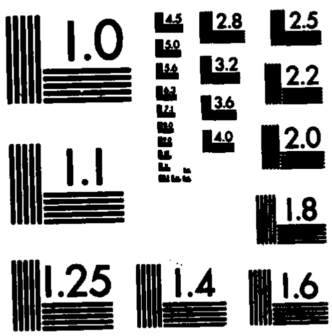

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Use of Radar Position Reports for Estimating Aircraft Acceleration

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Final Report

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16. Abstract

An inquiry was conducted into the possibility of calculating aircraft acceleration from radar position reports using second differences of range-bearing data. The analysis shows that data provided by the future monopulse radar receiver can be sufficiently accurate so that good estimates of acceleration may be calculated. Data from the present sensor system are of poorer quality and several additional scans of data must be included if the calculation is to be of value. In this case there will be a delay in estimating acceleration changes.

Observations of a B727 aircraft maneuvering at 425 knots and observed by a present generation radar at a distance of 100-140 nmi were analyzed. Root mean square heading errors were calculated to be about 5° for non maneuvering flight. Nine maneuvers, ranging in extent from 9° to 174°, were observed. Calculation suggest that these can be followed using acceleration calculations with an accuracy of about 10°. The peak heading error for one scan is estimated to be 32° for a right angle turn.

Although impractical for the present generation of ground based computer equipment acceleration estimates will be feasible when the next generation is in place. These estimates should be used with the future monopulse radar. Further analysis is required to determine if they should be used with the present radar.

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Note on Units

Units familiar to those with a background in navigation are generally used in this report. The nautical mile, abbreviated nmi, is the unit of length, the knot is the unit of velocity and the degree of arc, abbreviated "°" is the unit of angle. In many instances the metric equivalents of numerical values are noted in parentheses. The conversion factors are: 1 nmi = 1852 m, 1 knot = 0.514 m/s and 360° = 2π rad.

CONTENTS

1. INTRODUCTION

Over the course of the next twenty years planners expect the volume of air traffic to double. It is also expected that the size of the controller work force will remain essentially constant. If this scenario is realized then the nature of the controller's work will undergo a change. Necessarily more the controller's time will have to be devoted to the planning of orderly flight and less will be available for intense monitoring of the air situation. Consequently the controller must be expected gradually to place greater reliance on advanced automation functions as an aid to such monitoring.

From the point of view of safety the ultimate ground based automation function will be one similar to the presently installed conflict alert function. This function provides the final defense to midair collisions between two aircraft. It projects the calculated velocity vectors of an aircraft pair two minutes into the future and if this projection indicates that the pair will violate separation standards, currently 4.8 nmi (8.9 km) horizontally, then a message alerting the controller to the situation is displayed.

For proper operation the conflict alert function relies on the accuracy and proper processing of the aircraft position reports provided by the radar sensors (Reference 1).

Prediction of vertical and horizontal motion of aircraft is handled separately. In this report only horizontal motion will be considered. Processing of target reports (range, bearing) for horizontal motion at present provides for making a continuous estimate of position (two numbers) and velocity (two numbers) and correcting these according to the difference between the predicted location of the aircraft and its position as reported by the radar sensors. For the conflict resolution advisory function and conflict alert function the velocity estimate is of great importance:

$$\bar{v} = \bar{v}_1 + \beta \cdot (\bar{r} - \bar{r}_1)/T \qquad (1\text{-}1)$$

where $\bar{v}$, a vector, is the best estimate of aircraft velocities. As shown by the equation, it is calculated by adding to the previous estimate, $\bar{v}_1$, a fraction, β, of the discrepency between the expected position, $\bar{r}_1$, and the reported position, $\bar{r}$. T is the elapsed time between position reports.

The value of the feedback constant, β, depends on whether the aircraft is estimated to be in a maneuver (turning) or is cruising in a straight line. For maneuvers, in the en route system, β is taken as 5/32. For straight line flight the operational value of β is 3/64.

The velocity $\bar{v}$, of equation (1-1) is used as the predictor of future aircraft position. Such use is strictly correct only for the case of normal, non-banked, flight. When an aircraft is turning the direction, although not the magnitude, of the velocity vector is continually changing. There results a heading error in the en route system of from 30 degrees (0.52 rad) to 60 degrees (1.05 rad), the larger errors being associated with lower speeds (Reference 2).

To allow for banked, maneuvering motion it is desirable to replace expression (1-1) with one which includes an estimate of the acceleration experienced by an aircraft. Previous attempts in this direction accomplish this by augmenting the right side of (1-1) with the quantity

$$\gamma(\bar{r}-\bar{r}_1)\cdot t/T^2$$

where t is prediction time and the feedback parameter, γ, is zero except when the quantity $|\bar{r}-\bar{r}_1|$ exceeds a threshold (References 3 and 4).

Few attempts appear to have been made to measure turning acceleration directly, perhaps because of the limitations of available computer resources. In the future however, it appears these resources will be essentially unlimited (Reference 5). Under such circumstances the situation should be reviewed.

Acceleration can be estimated from noisy position reports to any precision desired provided that a sufficient number of reports is available. Operationally, however, it is important to have prompt estimates of any changes in acceleration. Consequently it is necessary to compromise between the conflicting requirements of precision and promptness. The analysis in section 2 of this report suggests that calculations using between three and four of the most recent reports of position are required in order to obtain a usable estimate of aircraft acceleration. Most of the uncertainty in the estimate is expected to be due to the comparatively poor accuracy of the bearing measurement.

Section 3 of this report, Observations, is devoted to a demonstration of the usefulness of direct calculation of acceleration when predicting aircraft position. Range/bearing data observed during a test flight involving a B727 aircraft are processed in such a way that the turning motion of the aircraft is observed correctly. Future motion of a turning aircraft can be predicted relatively accurately using the acceleration so measured.

The Discussion and Recommendations section of this report summarizes the investigation. Further study of the points eleborated in Sections 2 and 3 are indicated with a view towards implementing a tracking system suitable for use in the time frame 1988-2008.

2. ANALYSIS

An aircraft is moving with a constant angle of bank. Its velocity is $\bar{v}$, and its acceleration $\bar{a}$, where $\bar{v}$ and $\bar{a}$ are assumed to be two dimensional vectors. The curvature of the track is

$$|\bar{a} \times \bar{v}|/v^3 \qquad (2\text{-}1)$$

and the rate of heading change of aircraft heading is

$$|\bar{a} \times \bar{v}|/v^2, \qquad (2\text{-}2)$$

where $\bar{a} \times \bar{v}$ is the vector product of acceleration and velocity.

The value of the numerator in these expressions is given in the curvilinear coordinate system of the radar by the expression

$$r \dot{r} \ddot{\theta} - r \ddot{r} \dot{\theta} + 2 \dot{r}^2 \dot{\theta} - r^2 \dot{\theta}^3 \qquad (2\text{-}3)$$

where r is range, θ is bearing measured clockwise from the North direction and the "•" notation denotes the derivative with respect to time.

Radar sensors report range and bearing data at uniform intervals. These are usually twelve second intervals for the long range (200 nmi = 370 km) system and four second intervals for the short range (60 nmi = 111 km) system. For tracking with range/bearing data, it is useful to replace the differentials of expression (2-3) with the appropriate differences. The expression then becomes proportional to the following:

$$r_o(r_o-r_1)\Delta^2\theta - r_o(r_o-2r_1+r_2)(\theta_o-\theta_1) + 2(r_o-r_1)^2(\theta_o-\theta_1) - r_o^2(\theta_o-\theta_1)^3 \qquad (2\text{-}4)$$

where r_o, θ_o are the range and bearing of the latest report, r_1, θ_1 are those of the previous report r_2, θ_2 those of the second previous report, etc. $\Delta^2\theta$ is the second difference in bearing.

It will be assumed that the errors in range and bearing are statistically independent. In this case the variance associated with (2-4) is essentially composed of the terms

$$6 \,\mathrm{Var}(r)\, r_o^2\, (\Delta\theta)^2 + m \,\mathrm{Var}(\theta)\, r_o^2\, (\Delta r)^2 \qquad (2\text{-}5)$$

where Var(r) is the variance in range and Var(θ) is the variance in bearing.

The value of m in expression 2-5 depends on the number of position reports used in making the estimate of the second difference $\Delta^2\theta$.

Table 2-1

Estimating Second Differences of Bearing

Seven minimum variance formulas for second difference are listed. The variance coefficient, m in expression 2-5, decreases with the number of terms used in each formula but the responsiveness also decreases. In the last column, labelled Response, is entered the number of replies required before 80% of a change in curvature is measured.

Formula for $\Delta^2\theta$ (see footnote *)	Number of Terms	Variance Coefficient m	Response: Average Number of Replies required for 80% response
1,-2,1:1	3	6	1.6
1,-1,-1,1:2	4	1	2.1
2,-1,-2,-1,2:7	5	2/7=0.286	2.9
5,-1,-4,-4,-1,5:28	6	3/28 = 0.107	3.6
5,0,-3,-4,-3,0,5:42	7	1/21= 0.048	4.0
7,1,-3,-5,-5,-3,1,7:84	8	1/42 = 0.024	4.9
15,6,-1,-6,-9,-10 -9,-6,-1,6,15:429	11	2/429 = 0.005	6.9

* An example should make clear the meaning of the compact notation. By the third line is meant the following formula for $\Delta^2\theta$: $(2\theta_o-\theta_1-2\theta_2-\theta_3+2\theta_4)/7$ where θ_o is the value of the most recent report, θ_1 that of the next most recent report, θ that of the second most recent report, etc.

The use of additional reports, although providing a better estimate of the radius of curvature, results in additional delay in providing this measurement when the acceleration is changed. The situation is summarized in Table 2-1.

The expected performance of a tracker which estimates heading changes using second differences can be summarized by a formula for the expected variance in predicting heading change rate. For an aircraft cruising in the direction which makes the angle with a vector drawn from the sensor to the aircraft this formula may be written as follows

$$6 \frac{Var(r)}{(v\ \Delta t)^2} \frac{\sin^2 \alpha}{\Delta t^2} + m\ Var(\theta) \left(\frac{r}{v\ \Delta t}\right)^2 \frac{\cos^2 \alpha}{\Delta t^2} \qquad (2\text{-}6)$$

Formula (2-6) will be applied to six numerical examples: (1) en route traffic observed by the presently deployed Air Traffic Control Radar Beacon System (ATCRBS), (2) terminal traffic observed by ATCRBS, (3) en route traffic observed by a monopulse sensor of field quality, (4) terminal traffic observed by a monopulse sensor of field quality, (5) en route sensor observed by a monopulse sensor of laboratory quality and (6) terminal traffic observed by a monopulse sensor of laboratory quality.

Example 1: En route traffic, ATCRBS sensor. In this case typical numerical value associated with expression 2-6 are:

range, r:	100 nmi (185 km)
speed, v:	400 knots (206 m/s)
scan time, Δt:	12 seconds
Var(r):	1/64 nmi^2 (53,600 m^2)
Var(θ):	9x10^{-6} radian^2

If the value of m is taken as unity (Table 2-1), the standard deviation in heading change estimate, per scan, is 13 degrees (0.23 rad) independent of initial heading.

Example 2: Terminal traffic, ATCRBS sensor. In this case typical numerical values associated with expression 2-6 are:

range, r:	40 nmi (74 km)
speed, v:	200 knots (103 m/s)
scan time, Δt:	4 seconds
Var(r):	324x10^{-6} nmi^2 (1100 m^2)
Var(θ):	9x10^{-6} radian^2

If the value of m is taken as unity (Table 2-1), the standard deviation in heading, per scan is 32° (0.56 rad) for aircraft moving radially away from the sensor and 11° (0.19 rad) for aircraft moving transverse to the sensor.

Example 3: En route traffic, field quality monopulse sensor. This type of sensor provides an improved estimate of position (Reference 6). Typical values to be used in formula 2-6 are:

range, r: 100 nmi (185 km)
speed, v: 400 knots (206 m/s)
scan time, Δt: 6 seconds
Var(r): 26×10^{-6} nmi^2 (89 m^2)
Var(θ): 10^{-6} $radian^2$

If the value of m is taken as unity (Table 2-1), the standard deviation in heading measurement, per scan, is 9 degrees (0.15 rad) for aircraft moving radially away from the sensor and one degree (0.02 rad) for aircraft moving transverse to the sensor.

Example 4: Terminal traffic, field quality monopulse sensor. In this example typical values to be used with formula 2-6 are:

range, r: 40 nmi (74 km)
speed, v: 200 knots (103 m/s)
scan time, Δt: 4 seconds
Var (r): 26×10^{-6} nmi^2 (89 m^2)
Var (θ): 10^{-6} $radian^2$

If the value of m in Table 2-1 is taken as unity, then the standard deviation in estimating heading changes is 10 degrees (0.18 rad) per scan for aircraft moving radially away from the sensor and one degree (0.02 rad) per scan for aircraft moving transverse to the sensor.

Example 5: En route traffic, laboratory quality monopulse sensor. This type of sensor provides a greatly improved estimate of position (Reference 7). Typical value to be used in formula 2-6 are the same as those used in example 3 except:

Var(θ): 0.13×10^{-6} $radian^2$

If the value of m is taken as unity (Table 2-1), the standard deviation in heading measurement, per scan is 3 degrees (0.05 rad) for aircraft moving radially away from the sensor and one degree (0.02 rad) for aircraft moving transverse to the sensor.

Example 6: Terminal traffic with laboratory quality monopulse sensor. In this example typical value to be used with formula 2-6 are the same as those in used example 4 except:

Var(θ): 0.13×10^{-6} $radian^2$

If the value of m is taken as unity (Table 2-1), the standard deviation in estimating heading changes, per scan, is 4 degrees (0.07 rad) for aircraft moving radially away from the sensor and one degree (0.02 rad) for aircraft moving transverse to the sensor.

Typical aircraft turn rates vary between 20 degrees (0.35 rad) per scan for en route traffic and 12 degrees (0.21 rad) per scan for terminal traffic. The deviation in heading change measurements associated with laboratory quality monopulse sensors are expected to be considerably less than these rates. Hence the techniques described in this section for measuring turns are expected to be useful for predicting aircraft positions when reports of such quality are available.

The field quality monopulse sensors are permitted to have poorer bearing accuracy than the sensors tested at the FAA Technical Center and described herein as of laboratory quality. For flights cruising radially the deviation in heading computed above is comparable with the expected turn rates. Under these conditions one may still expect that the techniques described in this section will be useful but it may be found desirable to increase accuracy by allowing slower response times. This may be accomplished by inclu[illegible] more terms in the second difference calculation as described i[illegible] Table 2-1. However, a better solution, if feasible, is t[illegible]quire field sensors to provide a bearing accuracy comparable to [illegible]at observed during the tests at the Technical Center.

For ATCRBS sensors the calculated deviations are comparab[illegible] the expected turn rate. Under these circumstances the usefulness of the techniques discussed herein has been doubted (Reference 8). Nevertheless observations, as reported in the following section, suggest that in the case of traffic observed with the long range sensors, that aircraft turns can be reliably detected and reasonably well measured with these techniques.

Further discussion is reserved for section 4 beginning on page 13

3. OBSERVATIONS

3.1 General

An investigation of the validity of the formulas of section 2 and of the effectiveness of processing second differences to determine aircraft heading change was undertaken. This investigation was limited to en route traffic observed by a sensor of the Air Traffic Control Radar Beacon System (ATCRBS).

A B727 aircraft, cruising at 425 knots (219 m/s), followed a flight pattern which include a number of turns, all at a nominal 30 degree (0.52 rad) bank angle. The ATCRBS sensor was the operational radar located near Bedford, Virginia. The test flight was conducted in an area near Raleigh, North Carolina at distances between 100 nmi (185 km) and 140 nmi (260 km) from the radar.

The aircraft was under observation during 328 scans each of 12 seconds duration. A complete description of the flight is contained in reference 9. Range/bearing data from 140 of these 328 scans were selected for study. The purpose was to evaluate the usefulness of the ideas presented in section 2 when applied to en route traffic observed with the ATCRBS sensors. Cf. Example 1 on page 5.

The predicted heading change of aircraft motion is calculated from expression 2-4 which is written as $(v.\ \Delta t)^{-2}$ times the following sum of three terms

$$r_o(r_o-r_1)\Delta^2\theta + (r_o-r_1)^2(\theta_o-\theta_1) - r_o(r_o-2r_1+r_2)(\theta_o-\theta_1) \qquad (3\text{-}1)$$

In expression (3-1) r_o (θ_o) is the range (bearing) of the current target report, r_1 (θ_1) is the range (bearing) of the previous target report, r_2 (θ_2) is the range (bearing) of the second previous target reports, etc.

This expression was evaluated using three expressions to approximate the second difference in bearing, $\Delta^2\theta$. These approximations are selected from those described in Table 2-1 on page 4.

3.2 Observations when Aircraft is Not Maneuvering.

During twenty scans (240 seconds) the test aircraft cruised at a speed of 425 knots in a direction making an angle of 78 degrees (1.36 rad) with the vector to the sensor. The mean heading change and the variance in heading change varied with the second difference approximation as follows:

Second difference approximation, $\Delta^2\theta \approx$

Mean Error	2.3°	1.7°	1.6°
Standard Deviation	7.3°	4.3°	4.8°

It is notable that the standard observed standard deviation is considerably less than that which might be expected from the analysis of section 2.

During four scans the flight was generally headed towards the radar sensor. This is the most difficult flight geometry. The angle between the velocity vector and the direction to the radar was 24°. The mean errors in heading change and the standard deviation under these conditions were as follows:

Second difference approximation, $\Delta^2\theta \approx$

	$\theta_o-2\theta_1+\theta_2$	$(\theta_o-\theta_1-\theta_2+\theta_3)/2$	$(2\theta_o-\theta_1-2\theta_2-\theta_3+\theta_4)/7$
Mean Error	-1.0	-0.6	0.4
Standard Deviation	22°	9.9°	5°

3.3 Maneuvers

Nine maneuvers were observed during the course of the 140 scans. The data were reduced using the formula of section 2 (Table 2-1) with the results shown in Table 3-1. When a report was missing, as occurred seven times during the 140 scans of observation, the estimate of the second difference was made with whatever data were available. In particular the approximations

$$\Delta^2\theta \approx (\theta_o-3\theta_2+2\theta_3)/3$$
$$\text{or } (2\theta_o-3\theta_1+\theta_3)/3$$
$$\text{or } (\theta_o-\theta_1-\theta_3+\theta_4)/3$$
$$\text{or } (\theta_o-2\theta_2+\theta_4)/4$$

were used when appropriate. The exact approximation chosen depends on which report was missing. For the meanings of the symbols see page 3.

Table 3-1

Maneuver Detection Using Second Differences of ATCRBS Data

Estimate of change of heading calculated from formulas of section 2 using ATCRBS data. Comparison with plot data. B727 aircraft at range of about 120 nmi (222 km) cruising at 425 knots (219 m/s). Maneuvers flown generally at 30° (0.52 rad) nominal bank angle. Entries in degrees of arc.

	Extent of Maneuver			
		From expression 2-2, $\Delta^2\theta \approx$		
Scan Maneuver Starts	From Plot Data	$\theta_0+2\theta_1+\theta_2$	$(\theta_o-\theta_1-\theta_2+\theta_3)/2$	$(2\theta_o-\theta_1-2\theta_2-\theta_3+2\theta_4)/7$
421	89°	96°	96°	90°
435	31	37	95	57
440	9	0	0	0
446	42	35	31	61
459	48	28	36	54
474	41	54	50	44
485	87	101	84	84
513	-75	-64	-66	-69
522	174	211	200	174
Average Error:		3.3°	6.9°	4.1°
Root mean Square error:		16.6°	24.2°	11.6°

Results of the calculations with data reported from the observed maneuvers are summarized in the Table 3-1. The briefest of the nine maneuvers, one of 9 degrees (0.16 rad) was not detected using any of the means of maneuver detection based on expression 2-2 (page 3). The other eight maneuvers, of extent between 31 degrees (0.54 rad) and 174 degrees (3.04 rad), were all detected and measured with the accuracy shown in the Table.

Although the number of instances is too few to enable one to draw reliable statistical inferences, the average errors and the root mean square errors associated with the nine examples are shown in the bottom two rows of the Table. A positive average error means that the error in the calculation overestimates the extent of the maneuver.

The approximation with $\Delta^2\theta \approx (2\theta_0-\theta_1-2\theta_2-\theta_3+2\theta_4)/7$ was investigated from the point of view of response. The maximum error for each of the maneuvers is listed and the delay is measured from the scan when the maneuver starts, estimated by eye, to the scan with the maximum delay.

Scan Maneuver Starts	Maximum Lag (Lead)	Delay
421	32°	4 scans
435	(10)	2
440	9	Maneuver not detected
446	32	2
459	26	3
474	16	1
485	30	2
513	31	4
522	25 41	2

The maneuver beginning at scan 522 was an S-turn. The 41° lag was experienced on the second leg of the "S", two scans after the beginning of that part of the maneuver.

4. DISCUSSION AND RECOMMENDATIONS

4.1 Discussion

Proper operation of the conflict alert function or of any similar successor function requires generally only that aircraft velocity be measured accurately. The assumption of zero acceleration is usually the correct one in practise. However, although unusual, it is of course possible for one or both members of an aircraft pair to be flown in such manner that the zero acceleration assumption is invalid. In these cases the motion is curvilinear and predictions of location based on the zero acceleration approximation will be considerably in error.

A turning airplane with neutralized control surfaces will tend to continue in a circular path of constant radius until these surfaces are manipulated to adjust the radius of the circle. Hence, when an aircraft is turning, proper operation of the conflict alert function will require that acceleration be taken into account for making predictions of position. It is not enough merely to have the correct velocity as the direction of this velocity is constantly changing.

The analysis of section 2 suggests that acceleration will be calculable from aircraft position reports when the radar sensors are monopulse equipped. However the measurement of acceleration using the presently deployed sensors of the Air Traffic Control Beacon System (ATCRBS) will result in acceleration estimates which will be somewhat imprecise.

That analysis suggests that the precision of acceleration measurements using the ATCRBS data is expected to depend on the type of sensor and its application. When observing jet traffic, the long range sensors can be expected to measure heading changes with an accuracy of about 11° (0.19 rad) for each radar scan of twelve seconds. Accuracy of the heading change measurements made with the short range sensors when observing traffic at 200 knots can be expected to be poorer than this. A heading change accuracy of between 13° (0.23 rad) and 32° (0.56 rad) per scan is expected.

These estimates assume a tracker which calculates acceleration by processing the position reports of the most recent four scans of observation. The accuracy can be improved by including data from additional reports. If this procedure is followed, however, the responsiveness of the acceleration measurement to changes in acceleration will be correspondingly poorer.

The estimates of section 2 are confirmed fairly well using position reports of a B727 aircraft observed by an operational

ATCRBS sensor. Root mean square (rms) errors of about 5 degrees (0.09 rad) in heading were observed for that portion of the flight path where no acceleration was involved. When the aircraft made a slight maneuver of 9 degrees (0.16 rad) this maneuver was not observed. Eight other maneuvers, ranging in extent from 31 degrees (0.54 rad) to 174 degrees (3.04 rad) were observed and measured. The rms error of the extent of the nine maneuvers was calculated to be approximately 10 degrees (0.17 rad).

Accepting the analysis of section 2, it appears that when monopulse equipment is in place then the estimate of aircraft acceleration using second differences of reported position should be part of the software supporting the conflict alert function or whatever successor function is chosen. This conclusion appears inescapable since the best alternative involves peak heading errors of 20 degrees (0.35 rad) during maneuvers (Reference 4, Figure 3-53 et seq). It is prudent not to tolerate such errors if they can be avoided.

Two critical assumptions underlie the analysis of section 2. First, it is assumed that errors are statistically independent. Secondly, the variance in bearing is taken as 10^{-6} radian^2 or less. Both of these assumptions can be rendered invalid by premature truncation of the bearing datum when it is transmitted from the sensor to the processing equipment. In order to preserve a bearing variance of 0.13×10^{-6} radian^2 at least 14 bits, and preferably 16 bits, of information must be transmitted. If, as in the presently deployed system, transmission is limited to 12 bits, then the full potential of the monopulse sensor equipment for predicting aircraft motion cannot be realized.

Returning to the question of the presently deployed ATCRBS sensors the situation is not entirely clear. The rms errors of 5 degrees (0.09 rad) per scan in heading associated with the long range radar do not in themselves necessarily negate the use of acceleration estimates calculated from second differences. There are two mitigating factors to consider.

First, a "typical" heading change is about 15 degrees (0.26 rad) per scan so that the rms error of one third of this value may be practicable since false calls of turns should occur relatively rarely, about once per track hour. Such a false call rate may be tolerable operationally, depending on other details of the conflict alert algorithm which is adopted.

Secondly, the rms error can be reduced further by including additional position reports when estimating acceleration. Such an approach will have adverse effects on responsiveness but these effects may be tolerable.

A superficial review of the analysis suggests that poor results will be obtained when the reports from short range (60 nmi = 111 km) radar sensors are used for estimating acceleration. Heading errors (rms) of between 13 degrees (0.23 rad) per scan and 32 degrees (0.50 rad) per scan will be expected when using four position reports for making estimates. However, these short range radars employ the short scan time of four seconds and it is possible to reduce the expected errors by including additional, and earlier, position reports in the calculations.

Referring to table 2-1 on page 4, it is found that if delays in measuring heading changes of 20 seconds are tolerable then the calculated errors can be expected to be reduced to between 3 degrees and 6 degrees. Further reductions are possible if longer delays are permitted and the last entry in the table is probably near the limit of real possibilities. A delay of 32 seconds is expected in calculating heading changes when the complicated formula for including the 11 most recent position reports is used to estimate the second difference in bearing. Using 11, rather than 4 position reports, the standard deviation associated with this estimate is expected to be reduced by a factor of $\sqrt{429/2} = 15$.

Noting that aircraft observed by short range radars will ordinarily complete a right angle turn after about 28 seconds, typical turns will ordinarily be completed before a high accuracy estimate of heading change, one valid to within 2 degrees (0.037 rad), can be completed.

It appears that the desirability of using acceleration estimates in the prediction of aircraft notion cannot well be decided entirely from analyses based on the concepts presented thus far. The expected output from any processor of target reports is time - the time to violation of separation standards or the time to a midair collision. This time should ideally be a definite number, somewhat greater than two minutes, and any decrease in it due to processing shortcomings or sensor inaccuracies is to be avoided if possible.

The proper analysis must be based generally on a comparison of warning time to be expected in critical air situations. Accurate acceleration estimates and prompt response imply good warning times and conversly.

However, when estimating aircraft acceleration from radar position reports accuracy and promptness are mutually exclusive. There is the further complication that longer warning times, ordinarily desirable, will trigger unwanted alerts, those where projections based on the "true" velocity vectors of an aircraft pair do not violate separation standards but where heading uncertainties fool the automatic alerting software into declaring a nuisance alert.

As an example of the kind of analysis required consider how the conflict alert function protects airspace users. The velocity vector of the protected aircraft is projected 2 minutes into the future. The function has a radius of protection of 4.8 nmi (8.9 km). This means that if the direction of the velocity vector is in error by an amount greater than arctan (4.8/(vx2 minutes)) where v is aircraft speed in nmi per minute, then the aircraft is not protected against midair collision.

When an aircraft proceeding at 480 knots (247 m/s) enters a 90° (π/2 rad) turn, calculations show that it is unprotected for 110 seconds with the tracking algorithm presently used in the National Airspace System (Reference 10). A review of the 90° (π/2 rad) turns of the test aircraft, which cruised at 425 knots (219 m/s) shows that when acceleration is estimated from second differences, using the formulas of subsection 3.3 (page 8) that it is unprotected for 24 seconds during each turn. Thus the safety of a user will be increased in this case by a factor of four if a procedure similar to that described in section 2 of this report is adopted in place of the present one.

It will be noted from section 2 that the accuracy of heading estimates for aircraft depends on the flight geometry of the aircraft with respect to the sensor. If the aircraft aspect relative to the sensor is nose on or tail on then the heading estimate is comparatively poor.

Conversely, if the aircraft aspect is generally broadside to the sensor the heading estimate is comparatively good. This follows directly from the detection characteristics of radar sensors. Their accuracy in the radial direction is better than their accuracy in the transverse, so much so, that as in section 2, elaborate processing of the azimuthal data is required to make accurate second difference estimates. Two consequences follow.

First: If at all possible the bearing accuracy of the ATCRBS sensor should be improved. Any approach which reduces uncertainty by a factor of only a few would be a welcome improvement and one which can be used directly in improving heading change estimate (expression 2-6 on page 5). This improvement can be taken as either improved accuracy or improved response. Full monopulse processing of received signals (Reference 7) solves the problem but if any more economical solutions are available they should be reviewed.

Second: Other things being equal, the heading change of an aircraft should be determined from data provided by the sensor which views the aircraft generally broadside rather than from data provided by the sensor which looks nose on or tail on to the aircraft. See figure 4-1. Double radar coverage of aircraft is frequently available throughout the conterminous United States, particularly in airspace where aircraft are observed with the long range radar sensors.

The processing of radar sensor data as discussed in section 2 requires additional computer power beyond that presently devoted to such use. Essentially unlimited processing power is expected to be available in the immediate future at relatively moderate cost (Reference 5).

Aircraft acceleration can be determined in ways other than those described in this report. In particular it is possible to avoid the problem of azimuth accuracy by employing bistatic radar receivers. It is also possible to observe the aircraft range rate directly by processing phase information so that first differences in range rate can be substituted for second differences in range (Reference 9).

The subject has not been thoroughly explored as yet. The four recommendations which follow are expected to lead to worthwhile results.

4.2 Recommendations

Recommendation one: If feasible, improve beamsplitting techniques for the sensors of the presently deployed Air Traffic Control Radar Beacon System (ATCRBS). Aircraft bearing is the most difficult position datum to estimate.

Recommendation two: This recommendation applies only for monopulse receivers. Estimate aircraft acceleration by using second difference techniques (Section 2) or otherwise. Use these estimates for the short term prediction of aircraft motion in the conflict alert function. If second difference techniques are used at least 14 bits , and preferably 16 bits, of bearing information should be transferred from the sensor for use by the tracker.

Recommendation three: This recommendation applies only for ATCRBS receivers. Investigate the possibility of using estimates of aircraft acceleration using second difference techniques. This investigation should include the determination of system figures of merit (warning times, missed alerts, nuisance alerts), rather than geometric quantities for making evaluations.

Recommendation four: When an aircraft is in view of several sensors simultaneously, prefer the data from the sensor which views the aircraft broadside to those from the sensor which views it nose on or tail on.

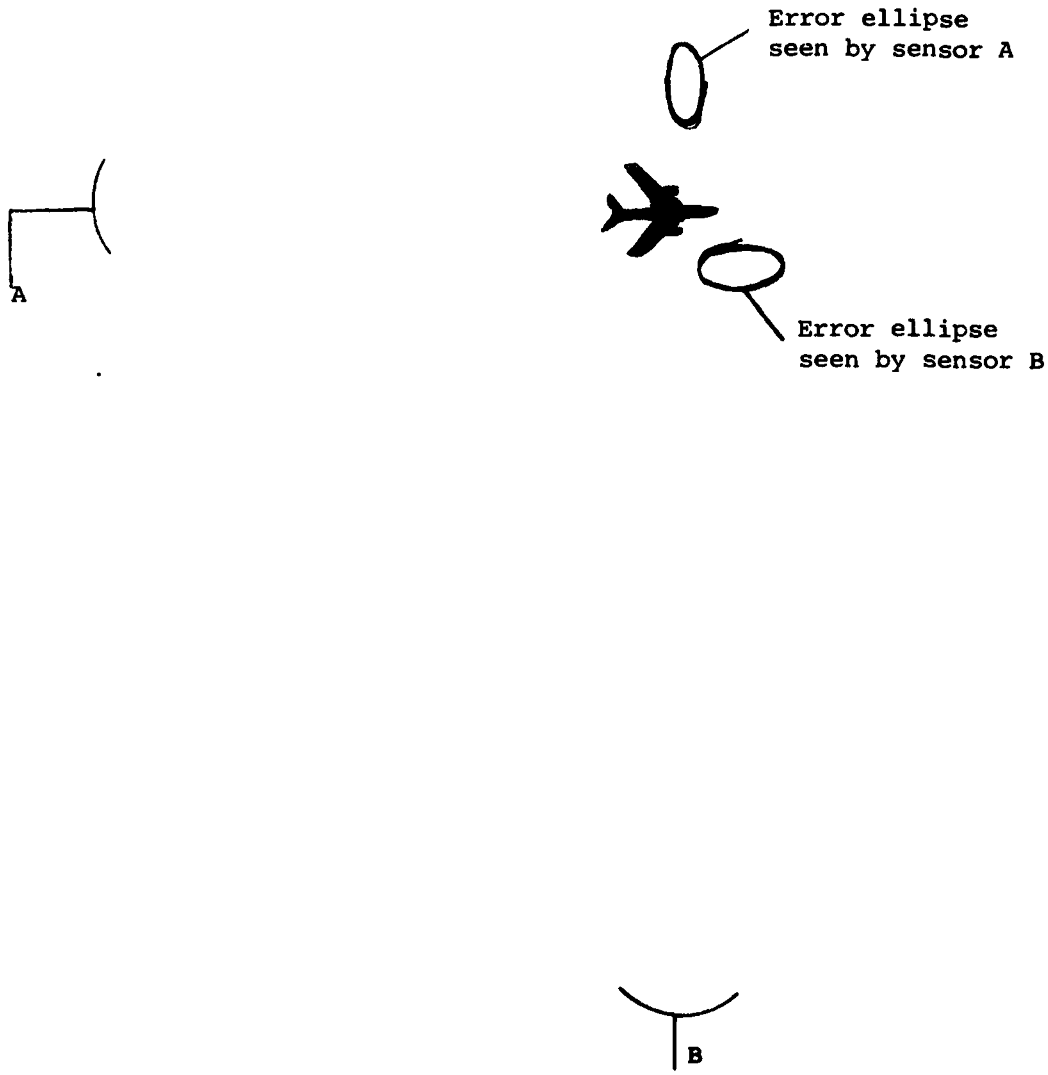

Figure 4-1. Aircraft proceeding generally East is observed simultaneously by both radar sensors A and B. Heading changes are more readily observable from the data provided by sensor B than from those provided by the A sensor. This is because the change in heading will affect principally the range measurements of sensor B and the bearing measurements of sensor A.

Range measurements are normally more accurate than bearing measurements, hence any change in heading will be more readily detected by the B sensor.

If these recommendations are followed one may expect improved performance of advanced air traffic functions, such as the conflict alert function, and consequently an improved level of safety for users of the National Airspace System.

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